Theoretical study of atoms dynamics in optical dipole trap

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ABSTRACT

Theoretical study and computer simulation results for stochastic dynamics of two atoms trapped in an optical dipole trap under action of a probe resonant radiation are presented. The radiation force correlations resulting from our model lead, in addition to cold collisions, to a tendency for atoms escape in pairs from the trap.

1. INTRODUCTION

Cold collisions between atoms in an atomic trap (magneto-optical trap or optical dipole trap) are thought to be one of the key mechanisms for atomic losses from the trap.[1] As it has been recently shown experimentally, this mechanism could result to a tendency for atoms escape in pairs (Fig. 1).[2,3] At the same time, another fundamental mechanism, long-range resonant dipole-dipole interactions (long-distance RDDI), also results in correlations between interacting atoms and therefore to the same tendency for atoms escape in pairs.[4] In this paper, we discuss this second mechanism.

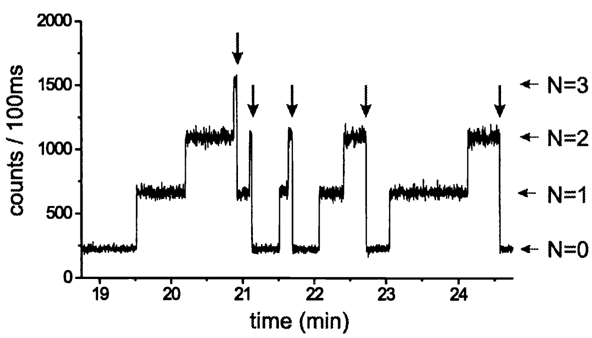

Figure 1. Excerpt from a typical fluorescence signal from a magneto-optical trap observed with an avalanche photodiode. Five isolated cold collisions (two-atoms losses) are indicated by arrows. (Picture is taken from Ref. 1.)

In our model, we consider dynamics of atoms in a red-detuned optical dipole trap.[5–7] To clarify a role of long-distance RDDI correlations we shine the atoms additionally by a probe (weak) resonant laser filed. This additional field enhances significantly interactions between atoms via jointly emitted photons and reveals in increasing correlations of stochastic radiation forces acting on atoms. For simulation of atomic dynamics we use quasiclassical approximation for the emission radiation force.

In computer simulation, we analyze the atomic dynamics in the trap and the corresponding atomic losses by varying the frequency detuning of the probe laser field and its intensity.

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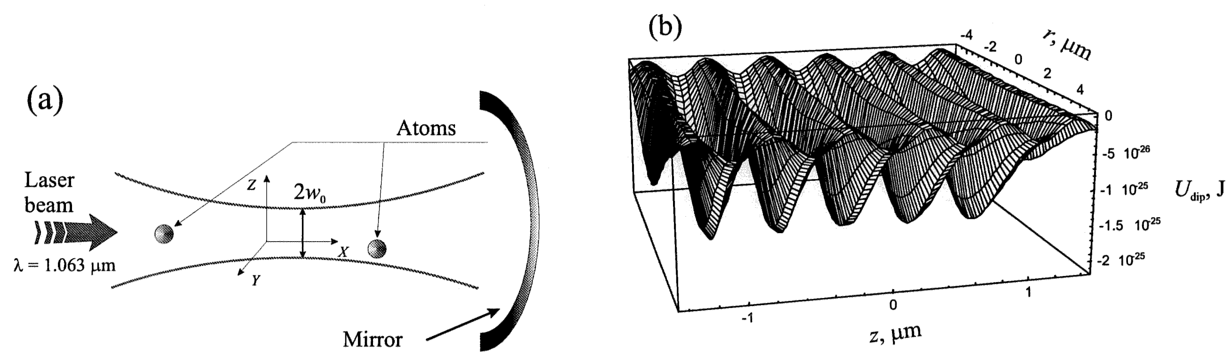

Figure 2. Scheme of a standing-wave optical dipole trap (a) and potential energy of the trap in radial and axial directions (b). The potential surface shows a regular pattern of micropotential holes.

2. MODEL OF OPTICAL DIPOLE TRAP

An optical dipole trap for neutral atoms can be made of a tightly focused powerful laser beam frequency of which is far-detuned from the working transition of trapped atoms. One can then easily show that interaction of the induced dipole moments of atoms in the trap with the inhomogeneous electric field along the beam profile lead to the restoring force, which "traps" the atoms inside the beam.[5] Surely, the described trapping mechanism works only for atoms with low energies (at temperatures of about mK) because the potential of the dipole trap is shallow. Therefore, atoms should be cooled down, for example in a magneto-optical trap, before loading into the optical dipole trap.[3,6,7]

In an experiment, for controlling positions of atoms in the trap it could be advantageous to keep single atoms in micropotential holes. Such a regular pattern of micropotential holes in the optical dipole trap potential can be formed with the use of counter-propagating laser beams, which form necessary structure due to the interferience. A scheme of a standing-wave optical dipole trap formed with the counter-propagating laser beams is shown in Fig. 2a. It uses only one laser beam, which interferes with the beam reflected from the mirror that preserves the wave front and the polarization.[8] The potential of the trap is shown in Fig. 2b.

Parameters of the optical dipole trap we used in our model for numerical simulations have been taken similar to the parameters of the experimental setup of Refs. 6,7 Namely, we consider the optical dipole trap formed by focusing 2.5 W Nd:YAG laser beam (1.063 μm) with linear polarization along x-axis into an area with diameter of about 5 μm. Cesium atoms trapped in the optical dipole trap we assume two-level atoms with the lifetime of the excited state $1/\gamma = 3.07 \times 10^{-8}$ s and the dipole moment of the working transition $d = 8.01 \times 10^{-18}$ CGSE. Frequencies of atomic oscillations in radial and axial directions are equal to $\omega_r \approx 60$ kHz and $\omega_z \approx 1.5$ MHz, respectively.

In the following, we will consider the red-detuned optical dipole trap configuration. Such a trap is formed by a laser beam tuned far below the atomic resonance frequency. We will also assume that the laser beam with power P forming the trap has the Gaussian intensity profile:

$$I(r,z) = \frac{2P}{\pi w^2(z)} \exp\left(-2\frac{r^2}{w^2(z)}\right) \cos^2(kz), \tag{1}$$

where r is the radial coordinate and the half-waist beam diameter $w(z)$ depends on the axial coordinate z as

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2}. \tag{2}$$

Here w_0 is the beam waist diameter and $z_R = \pi w_0^2/\lambda$ is the Rayleigh length.

As we mentioned earlier, the thermal energy $k_B T$ of the atomic ensemble should be much smaller than the potential depth of the trap and, therefore, movement of the trapped atoms in radial direction is reasonably small compared to the beam waist diameter and in the axial direction movement of atoms is smaller than the Rayleigh length. In this case, the optical potential of the trap can be approximated as:[5]

$$U_{\rm dip}(r,z) = -U_0\cos^2(kz)\left[1-2\left(\frac{r}{w_0}\right)^2-\left(\frac{z}{z_R}\right)^2\right]. \tag{3}$$

The oscillation frequencies of the trapped atoms are equal to $\omega_r = (4U_0/mw_0^2)^{1/2}$ in radial direction and $\omega_z = k(2U_0/m)^{1/2}$ in axial direction, so that the potential energy reads then as

$$U_{\rm dip}(r,z) = \frac{(n_1-n_2)\pi c^2\gamma}{2\omega_0^3}\left(\frac{2}{\Delta_2}+\frac{1}{\Delta_1}\right)I(r,z), \tag{4}$$

where $(n_1 - n_2)$ is the difference between population of the low and upper atom levels, $\Delta_2 = \omega(^2\mathrm{P}_{3/2} \rightarrow ^2\mathrm{S}_{1/2})$ and $\Delta_1 = \omega(^2\mathrm{P}_{1/2} \rightarrow ^2\mathrm{S}_{1/2})$ are the transition frequencies.

3. MODEL FOR THE LONG-RANGE RESONANT DIPOLE-DIPOLE INTERACTIONS

We assume that interaction of atoms trapped in the trap is the long-range RDDI, which is governed by stochastic process that leads, in its turn, to a stochastic radiation force between atoms. Then, we suppose that the laser detuning $\Delta \ll \omega_0$ and atoms are located at a distance of the order of laser wavelength, $R_{12} \gtrsim \lambda$. With these assumptions, we can calculate the spectrum of atomic radiation force, keeping in mind that the dipole radiation of atoms for this purpose can be approximated with a white noise. Spectra N_{11} and N_{22} of the interaction force fluctuations between two atoms are equal and spectrum of single atom fluctuations takes the form:

$$N_{11} = N_{22} = \frac{\hbar\omega_0^5 d^2}{2\pi c^5}\langle\Delta\sigma_1^-\Delta\sigma_1^+\rangle I_{11}, \tag{5}$$

$$N_{12} = N_{21} = \frac{\hbar\omega_0^5 d^2}{2\pi c^5}\langle\Delta\sigma_1^-\Delta\sigma_2^+\rangle I_{12}, \tag{6}$$

where $d = \left(3\hbar c^3\Gamma/4\omega_0^3\right)^{1/2}$ is the transition dipole moment and $\hat{\sigma}_\pm$ are the atomic transition operators. For the case of dipole moments parallel to each other and orthogonal to the vector of displacement we have

$$I_{12} = \pi\begin{pmatrix}\tilde{I}_1 & 0 & 0\\ 0 & \tilde{I}_2 & 0\\ 0 & 0 & \tilde{I}_3\end{pmatrix},\quad I_{11} = \pi\begin{pmatrix}\frac{8}{15} & 0 & 0\\ 0 & \frac{16}{15} & 0\\ 0 & 0 & \frac{16}{15}\end{pmatrix}, \tag{7}$$

where

$$\tilde{I}_1 = \frac{4(9-\varphi_{12}^2)\cos\varphi_{12}}{\varphi_{12}^4} - \frac{4(9-4\varphi_{12}^2)\sin\varphi_{12}}{\varphi_{12}^5}, \tag{8}$$

$$\tilde{I}_2 = \frac{4(3-\varphi_{12}^2)\cos\varphi_{12}}{\varphi_{12}^4} - \frac{4(3-2\varphi_{12}^2)\sin\varphi_{12}}{\varphi_{12}^5}, \tag{9}$$

$$\tilde{I}_3 = \frac{-4(12-3\varphi_{12}^2)\cos\varphi_{12}}{\varphi_{12}^4} + \frac{4(9-12\varphi_{12}^2+\varphi_{12}^4)\sin\varphi_{12}}{\varphi_{12}^5}, \tag{10}$$

$\langle\Delta\sigma_1^-\Delta\sigma_2^+\rangle$ is the correlation function of the transition operator that reads

$$\langle\Delta\sigma_1^-\Delta\sigma_2^+\rangle = \frac{-4g\tilde{g_L}^4(1+4\delta^2)}{\left[(1+g)^2+4(\tilde{g}_L^2+\delta^2)+4(1+g)^2\delta^2+4(\tilde{g_L}^2+2\delta^2)^2\right]^2}. \tag{11}$$

Here $\tilde{g_L} = g_L/\Gamma$ is the dimensionless Rabi frequency, $g_L = Ed/\hbar$ is the Rabi frequency, $\delta = \Delta/\Gamma$ is the dimensionless laser detuning, and

$$g = \frac{3}{2}\frac{\varphi_{12}\cos\varphi_{12} - \sin\varphi_{12} + \varphi_{12}^2\sin\varphi_{12}}{\varphi_{12}^3}. \tag{12}$$

The correlation function for fluctuations of a single atom in the trap, $\langle \Delta\sigma_1^- \Delta\sigma_1^+ \rangle$, can be written as

$$\langle \Delta\sigma_1^- \Delta\sigma_1^+ \rangle = \frac{2\tilde{g_L}^4 \left[1 + 2(\tilde{g_L}^2 + \delta^2)^2 + g^2(1 + 4\delta^2)\right]}{\left[(1+g)^2 + 4(\tilde{g_L}^2 + \delta^2) + 4(1+g)^2\delta^2 + 4(\tilde{g}_L^2 + 2\delta^2)^2\right]^2}. \quad (13)$$

To clarify the role of the long-range RDDI we suggest to shine atoms in the optical dipole trap with an additional weak laser field tuned into the resonance with the atomic transition. This will increase the RDDI between atoms and, therefore, lead to the increase in fluctuation amplitude of each atom in the trap. Varying the laser detuning δ of the weak laser field and its intensity we can control the radiation force between atoms.

4. MODELING OF ATOMS DYNAMICS IN THE TRAP

Motion of atoms in the trap is governed by the following equation:

$$m\ddot{\mathbf{r}}^{(k)} = -\frac{\partial U_{\text{dip}}^{(k)}}{\partial \mathbf{r}} + \mathbf{F}_{\text{cool}}^{(k)} + \mathbf{F}_{\text{RDDI}}^{(k)}, \quad k = 1, 2, \quad (14)$$

where index k numbers the atoms in the trap, U_{dip} is the trapping potential, $\mathbf{F}_{\text{cool}}^{(k)}$ is the cooling force (linear friction), and $\mathbf{F}_{\text{RDDI}}^{(k)}$ is the fluctuation force due to the RDDI. The cooling force can be written as[9]

$$\mathbf{F}_{\text{cool}} = -m\alpha\dot{\mathbf{r}}_z, \quad (15)$$

where

$$\alpha = -\frac{2\hbar\omega^2}{mc^2}\frac{I}{I_0}\frac{2\delta/\gamma_0}{[1 + (2\delta/\gamma_0)^2]^2},$$

I is the intensity of the weak laser field used to enhance the RDDI interactions in the trap, and I_0 is the saturation intensity of the atomic transition.

The fluctuation force $\mathbf{F}_{\text{RDDI}}$ can be modeled with a white noise because the correlation time, determined by the radiation decay rates, is small in comparison with all other time scales of atom's dynamics in the trap. The joint correlation matrix for the fluctuating force $\mathbf{F}_{\text{RDDI}}$ has the form:

$$N = \begin{pmatrix} N_{11} & -N_{12} \\ -N_{12} & N_{11} \end{pmatrix},$$

where N_{11} and N_{12} are determined by Eqs. (5), (6). In accordance with Eq. (7), the only correlated projections in this matrix are because of the N_{12} matrix element. The corresponding correlated pair of random values x_1, x_2 can be determined then as

$$x_1 = \sqrt{\frac{6(N_{11}+N_{12})}{\Delta t}}\xi_1 - \sqrt{\frac{6(N_{11}-N_{12})}{\Delta t}}\xi_2,$$
$$x_2 = \sqrt{\frac{6(N_{11}+N_{12})}{\Delta t}}\xi_1 + \sqrt{\frac{6(N_{11}-N_{12})}{\Delta t}}\xi_2,$$

where Δt is the time step in the course of computer simulation. Here ξ_1, ξ_2 are the independent random variables uniformly distributed in the range $(-1/2, 1/2)$, so that we can simply replace $x_i\Delta t$ with the integral $\int_{\Delta t} x(t)dt$, which gives us correct dispersion $N\Delta t$ for the white noise.

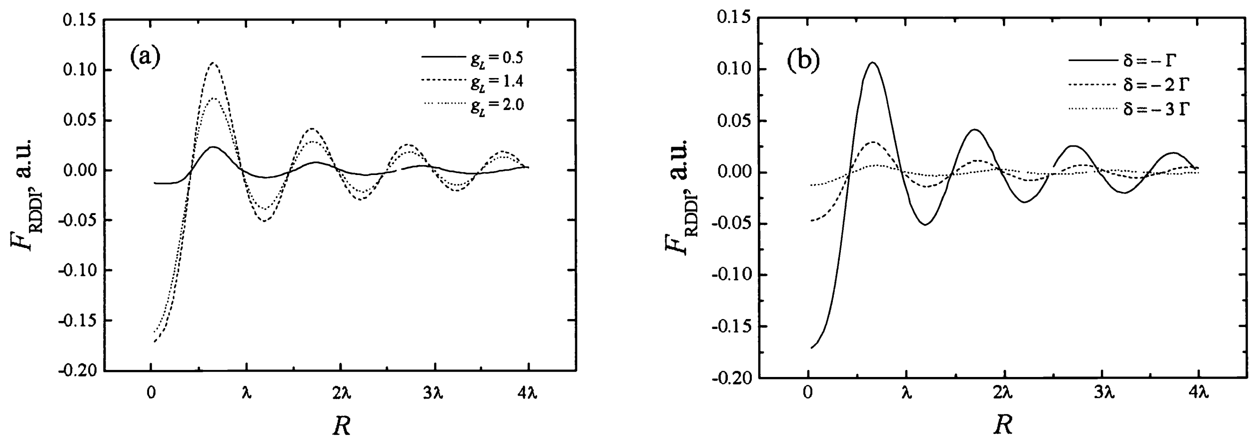

Figure 3. The RDDI force between atoms versus interatomic distance (in wave lengths) for different values of the dimensionless Rabi frequency g_L (a) and laser detuning δ (b).

5. COMPUTER SIMULATION RESULTS

In computer experiment, we analyze the role of interaction of atoms within large distances controlling the dynamics of atoms in the trap, which depends on the intensity and detuning of the probe laser beam. Influence of a buffer gas and cold collisions with the buffer gas atoms in the trap have not been taken into account in our model—we plan to do that next. We can neglect cold collisions between atoms trapped in the micropotential minima of the optical lattice in the trap.[10]

We will start discussion of computer simulation results for atoms dynamics in the trap with the RDDI force acting between atoms. Its oscillating dependency on the distance between atoms is shown in Fig. 3. The atoms in the trap are trapped in the micropotential minima of the optical lattice with the period of $\lambda/2$, where λ is the wavelength of the laser beam forming the optical dipole trap (and the lattice). As it follows from Fig. 3, the RDDI force for atoms strictly located in the micropotential minima positions is equal to zero. This means that the RDDI interaction between trapped atoms is only due to the atoms fluctuations (oscillations) around their equilibrium positions in the micropotential minima. Also, the effective RDDI force amplitude decreases with increasing the interatomic distance.

Modeling the trajectory of atoms in the trap with the use of Eq. (14) we set initial coordinates of the atoms as random deviations from the equilibrium positions in the minima of the micropotentials and their initial velocities distributed according to the Maxwell distribution. The latter depends on the temperature of atoms, which we set equal to the experimental value of 40 μK (see Ref. 6,7). At this temperature atoms are localized in the micropotential minima for quite a long time.

It is also necessary to estimate if the probe laser beam shining the atoms in the trap could kink them out of the micropotential minima. For this we can estimate the energy, which is transferred to an atom by a photon from the probe laser beam, that reads

$$E = \frac{(\hbar\omega/c)^2}{2m} \simeq 2 \times 10^{-2}\,\mathrm{K}. \tag{16}$$

This number does not exceed the depth of a potential hole of the trap in the center of the beam and, therefore, the photons from the probe laser beam will not kink the atoms out from the micropotential minima with the 100% probability after a single photon emission. This enables us to use in our calculations a diffusion approach for the process of atoms radiation escape.

Spectra of fluctuations of atoms in the micropotentials of the optical lattice are shown in Fig. 4. The one-atom spectrum is essentially enriched for the case of two interacting atoms with respect to the case of a single atom, especially in the low frequency region. This means, that the correlation between two $\mathbf{F}^{(k)}_{\mathrm{RDDI}}$ forces ($k = 1, 2$) in Eq. (14), which is the only mechanism coupling the atoms, plays an essential role in atoms stochastic dynamics. If there is no correlation ($g = 0$), the dynamics of every atom is independent from each other and corresponding

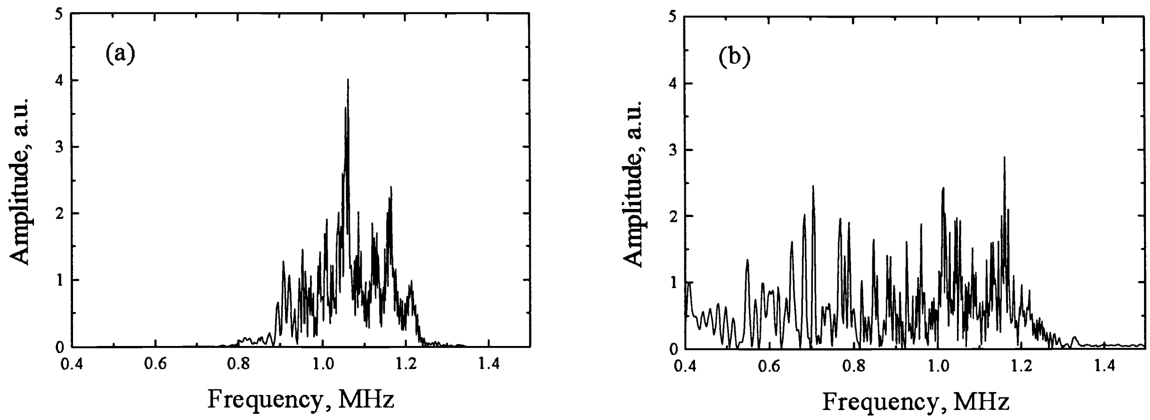

Figure 4. Spectra of fluctuations of a single atom (a) and one of the two atoms in the trap (b) along z-axis.

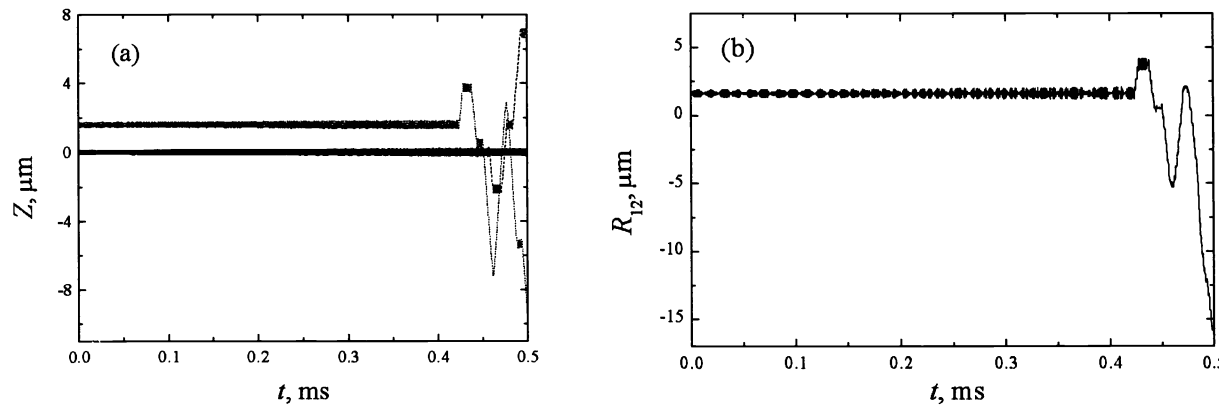

Figure 5. a) Temporal dependence of the axial coordinate of a single atom (solid line) and two atoms in the trap (dotted and dashed lines for each atom, respectively) with the same initial conditions. b) Temporal dependence of the axial projection R_{12} of the distance between two atoms in the optical dipole trap.

spectrum is shown in Fig. 4a. Having fluctuation forces depending on both atoms coordinates (see Eqs. (6), (11)), we get an essential modifications of their dynamics. Interaction via common photons emitted by the two atoms provides both correlation of their dynamics and its modification. Actually, the effect of a correlated escape of atoms in pairs from the trap shown in Fig. 5 can be associated with the nonzero fluctuation spectrum intensity at zero frequency, keeping in mind that the latter can correspond to the presence of zero center-of-mass velocity contribution in the total spectrum of an atom.

Fluctuations of a single atom also lead to the escaping of atom from the trap. That is why for the studying of fluctuation power of interaction between atoms it is necessary to compare the dynamics of two atoms with the dynamics of single atom with the same initial conditions. As a result, we can determine how one atom affects the dynamics of another atom. For this purpose, we model the dynamics of a single atom and two atoms in the trap with the same initial conditions and with the same fluctuation statistics of a single atom (Fig. 5). Analyzing atoms dynamics in the trap, we can see that both atoms leave the trap simultaneously (Fig. 5a).

Simulating the dynamics of two atoms in the trap we can quantitatively and qualitatively compare the trajectories of a single atom and one of the two atoms in the trap at any moment of time. This enables us to analyze the interaction between atoms long before the atoms escape from the trap. Analyzing, for example, temporal dependency of total energy of a single atom and one of the two atoms in the trap (Fig. 6) one can conclude that the increase of energy of one of the two interacting via RDDI atoms in the trap by contrast with more or less a monotonous dependency of a single atom in the trap is a sign of escaping atoms from the trap. Surely, decreasing time along which we observe

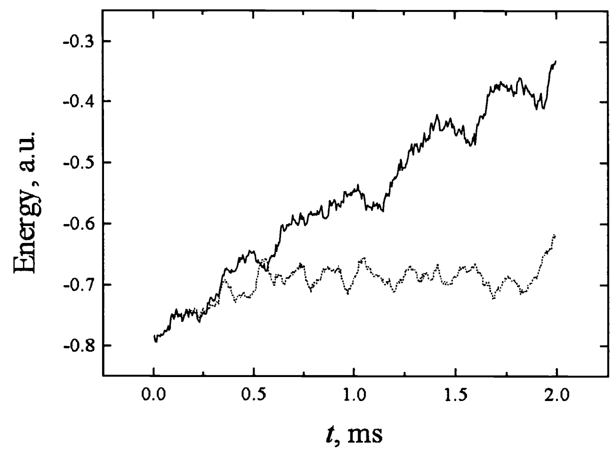

Figure 6. Temporal dependency of total energy of a single atom (dotted line) and one of the two atoms (solid line) in the trap. Initial conditions for the atoms are the same.

the atoms dynamics in a computer experiment, we decrease the accuracy of of our resulting conclusions. Therefore, special efforts have been made in our computer simulations towards selection of optimal parameters, namely the number of iterations in time, the time interval, an others.

6. CONCLUSIONS

In conclusion, theoretical study and computer simulation results for stochastic dynamics of two atoms trapped in an optical dipole trap under the action of a probe resonant radiation are presented. It is shown that interaction of atoms via common emitted photons reveals in an essential modification of the atoms dynamics. This modification leads to the enriched spectrum of atoms fluctuation dynamics and pair-correlated radiative escape. The model discussed here can be successfully used for analyzing atoms dynamics in optical dipole traps.

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