

The Information Capacity of the Λ -System–Photon Field Channel

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Received June 19, 2000

Abstract—The potentially attainable information capacity of a radiatively stable Λ -system, viewed as the input of a quantum information channel, is studied. The output of the channel is formed by the states of the photon field with optical excitation frequencies, which are created as a result of Raman pumping. The analysis is based on the notion of coherent information.

1. INTRODUCTION

For the experiments in the newest fields of physics—quantum computing, quantum communication, and quantum cryptography [1, 2]—the quantitative analysis of the possibly available amount of the specifically quantum *coherent information* [3, 4] is of substantial interest, since this parameter determines the potential information content of the obtained data. We have previously shown [5] how the general definition of the coherent information can be applied to the analysis of various physical models of quantum channels, which in general may have qualitatively different structure of the input and output state spaces.

In this work, we consider another important example of a quantum channel: the Λ -system [6] interacting with a free-space photon field. Atomic Λ -systems are particularly interesting for experimental implementations of the quantum-computing operations since the quantum information in this system can be stored in radiatively stable two-level subsystems of separate atoms and manipulated via radiatively excited states with the help of laser radiation [7]. The specific feature of the channel considered in this paper is the absence of any information at the output in the absence of laser excitation, which would otherwise act on the input states and lead to the excitation of radiatively active states. Thus, one can distinguish a special class of *active* channels whose capacity is intrinsically determined by the external disturbance. The examples of quantum channels considered in [5] include the following channels belonging this class: (i) a channel between an allowed transition and a forbidden transition of a (hydrogen) atom that are coupled by an applied external electric field and (ii) a channel between a pair of two-level atoms (TLAs) in the presence of an external excitation that entangles their states. The remaining examples belong to the class of *passive* channels, in which the information exchange takes place in the absence of an external perturbation as well.

2. THE DEFINITION AND THE PHYSICAL MEANING OF COHERENT INFORMATION

Consider a quantum channel whose input state is described by a density matrix $\hat{\rho}_{\text{in}}$ and that corresponds to a superoperator transformation \mathcal{C} , which relates the input and output density matrices, $\hat{\rho}_{\text{out}} = \mathcal{C}\hat{\rho}_{\text{in}}$. In this case, the coherent information is defined as

$$I_c = S[\hat{\rho}_{\text{out}}] - S_e. \quad (1)$$

Here, $S[\hat{\rho}] = -\text{Tr} \hat{\rho} \log_2 \hat{\rho}$ is the von Neumann entropy, which generalizes the classical Boltzmann–Shannon entropy $S[P(x)] = -\sum P(x_i) \log_2 P(x_i)$ of a random variable $x = \{x_1, x_2, \dots\}$ with the probability distribution $P(x) = \{P(x_1), P(x_2), \dots\}$; S_e is the so-called entropy exchange with the reservoir responsible for the noise in the channel. This definition is a direct generalization of classical Shannon’s information measure [8] to the channels whose both inputs and outputs are quantum.

In the case of the error-free transmission of all M possible values of a quantity x , the classical Shannon information is given by $I = \log_2 M$. For this choice of the logarithm base, a “bit” is conventionally introduced as a unit of measure. If the transmitted values of x have unequal probabilities described by the probability distribution $P(x)$, the above definition applies not directly to x , but rather to an ergodic sequence $x_k, k = 1, \dots, n$ of statistically independent copies of the variable x with the probability distribution $P(x_1) \cdot \dots \cdot P(x_n)$. In this case, as $n \rightarrow \infty$, the set of sequences with asymptotically nonzero probabilities consists of $M_n = 2^{S(P)}$ approximately equally probable members; the information per letter is therefore $(\log_2 M_n)/n = S(P)$. This result, which, in particular, plays a fundamental role in statistical physics, makes it possible to ascribe the amount of information $I = S(P)$ to the noiseless transmission of all possible values of the variable x with the

probability distribution $P(x)$. If errors may occur during the transmission, the corresponding nontrivial information-transmission channel is described by the conditional probability distribution $P(y|x)$ of the output variable y for a fixed input x . In this case, for long ergodic sequences, the accurately transmitted information per letter is given by Shannon's *mutual information*

$$\begin{aligned} I &= S(P_x) + S(P_y) - S(P_{xy}) \\ &= S(P_y) - \sum_x S[P(y|x)]P(x). \end{aligned} \tag{2}$$

Here, P_x , P_y , and P_{xy} denote respectively the probability distributions for the input x , the output y , and the pair x, y . The first equality in (2) demonstrates the symmetric (mutual) nature of the Shannon information with respect to the input and the output. The second equality represents the information as the difference between the entropy of the output variable y and the average entropy of the variable y introduced by the channel during the transmission of a given letter x . The meaning of the last equality is most evident for the channel, in which the transmitted letters x are mapped to nonoverlapping subsets M_x of the set of possible values of $y \in \bigcup M_x$, i.e., the distortions are reduced to the scatter of the output variable y within the regions M_x . The transmitted information is defined in this case as the difference between the full entropy of the variable y and the average entropy of the subsets M_x .

The basic definition of the *coherent information* is given by the relationship $I_c = \log_2 \dim H$, where H is the Hilbert space of the input quantum system all the states of which are transmitted without errors. The only name for the unit of quantum information is provided by the term "qubit," which is used in quantum computing theory and which corresponds to a two-level quantum system with the dimensionality $\dim H = 2$. The essentially new element of this theory is the quantum nature of the transmitted data, which can be found in any coherent superposition of basis elements. Using the same line of reasoning as in the previous paragraph, one can show that, if the distribution of the input states is described by the density matrix $\hat{\rho}_{in}$ and the channel transmits quantum states $\psi \in H$ noiselessly, a measure of quantum information is provided by the von Neumann entropy, i.e., the direct operator generalization of the expression for the classical entropy. The simplest channel realizing noiseless information transmission is, for example, the dynamic quantum evolution of a closed system between two time instants, $t = 0$ and $t \cong 0$.

If the quantum channel is noisy, the output state is a linear transform of the input state: $\hat{\rho}_{out} = C\hat{\rho}_{in}$. The role of the channel superoperator C is then analogous to that of the aforementioned conditional probability distribution $P(y|x)$ of a classical channel. The quantum gener-

alization of Shannon's definition (2) is based on the expression behind the second equality, the first term of which—the quantum entropy of the output—has a unique quantum generalization in the form of the von Neumann entropy. The second term, which describes the entropy introduced by the channel (the so-called entropy exchange S_e), should be zero for the error-free transmission, i.e., for the identity superoperator $C = I$, in the quantum case as well. On the other hand, if the input state is pure (the analog of a determinate classical state), this term must coincide with the output entropy, which in this case is completely due to the channel. These requirements can be satisfied if, instead of the input quantum system, one considers its extension $H \otimes H'$, where the degrees of freedom of H' do not interact with those of the channel, and the state $\hat{\rho}_p$ of the joint system is pure and such that its averaging produces the original state $\hat{\rho}_{in}$ [3]. This procedure of replacing the initial quantum system is termed the purification of a mixed quantum state. The corresponding transformation that the channel performs on the extended quantum system is $C \otimes I$, where I ensures that the variables of the ancillary system remain unchanged. The resulting entropy exchange then coincides with the entropy of the transformed extended system. The explicit form of the purified state in $H \otimes H$ (i.e., in the case $H' = H$) is implicitly contained in an expression derived in [4], which yields

$$\hat{\rho}_p = \sum_{ij} \sqrt{p_i p_j} |i\rangle \langle j| \otimes |i^*\rangle \langle j^*|, \tag{3}$$

where p_i , $|i\rangle$, and $\langle j|$ are respectively the eigenvalues, and the right and left eigenvectors of the density matrix $\hat{\rho}_{in}$; $|i^*\rangle$ and $\langle j^*|$ stand for the complex-conjugate vectors. The purified state is thus combined from the input state and its "mirror reflection" [9]. The corresponding entropy exchange is

$$S_e = S(\hat{\rho}_\alpha), \tag{4}$$

where

$$\hat{\rho}_\alpha = (C \otimes I)\hat{\rho}_p. \tag{5}$$

In the general case, the channel transformation C can describe the information transfer to an output system with a nonidentical Hilbert space, $H_{out} \neq H$.

In view of possible physical applications, we consider it important to give an adequate physical interpretation of the density matrix (5) introduced in [4], as well as the density matrix of the purified state (3) introduced here. Both of these definitions are based on the aforementioned mathematical arguments. Expression (3) describes the joint state of the input-mirror reflection system, which gives rise to the input-output quantum-mechanical state after the transmission. In the classical theory, the state (5) corresponds to the conditional distribution $P(y|x)$ of the output for a fixed input

and, at the same time, to the averaging over the input states in accordance with the distribution $P(x)$. The conditional distribution is represented by the superoperator \mathcal{C} , whereas the averaging over the input is represented by the structure of the wave function $\Psi_p = \sum \sqrt{p_i} |i\rangle |i^*\rangle$ that corresponds to the purified state (3). This two-particle state is entangled, that is, it cannot be decomposed as a statistical mixture of density matrices of the type $|\psi_i\rangle\langle\phi_i| \langle\phi_i|\langle\psi_i|$, which correspond to pure direct products $|\psi_i\rangle\langle\phi_i|$ of one-particle states. Its purely quantum fluctuations reproduce the mixed-nature fluctuations of the density matrix $\hat{\rho}_{in}$, which is defined in the first of the subspaces involved in the direct product $H \otimes H$. Thus, the density matrix (5) describes the state of the input–output system where the input is replaced by its mirror reflection [9]. It defines the entropy exchange of the channel and, according to its physical meaning, is qualitatively different from the usual one-instance density matrix since the corresponding nonzero entropy appears as a result of the transformation of the input state during its transmission in the channel. Unlike the usual two-particle density matrix, this matrix always corresponds to a pure state and zero entropy if the channel is noiseless.

3. THE MODEL OF THE CONSIDERED CHANNEL

The total dynamic system involved in the considered problem consists of a Λ -system, which, in turn, is constituted by three levels $|1\rangle$, $|2\rangle$, and $|3\rangle$, and the field of free photons described by the Hilbert space H_F . The corresponding total Hilbert space has the form $H_\Lambda \otimes H_F$, where H_Λ denotes the three-dimensional Hilbert space of the Λ -system. For the purposes of the most reliable storage and efficient manipulation of quantum information, the most interesting systems are those where the transitions $|1\rangle \longleftrightarrow |3\rangle$ and $|2\rangle \longleftrightarrow |3\rangle$ are allowed and the transition $|1\rangle \longleftrightarrow |2\rangle$ is forbidden. The set of radiatively stable states $|1\rangle$ and $|2\rangle$ can then be viewed as a *ground* two-level state (GTLS) of a Λ -system, which constitutes a qubit of quantum information. To extract this information in the form of photon field excitation, two laser fields, $E_L \cos(\omega_L t + \phi_1)$ and $E'_L \cos(\omega'_L t + \phi_2)$, are used, with their optical frequencies ω_L and ω'_L satisfying the Raman resonance condition $\delta_R \approx 0$ [6], where

$$\delta_R = \omega'_L - \omega_L - \omega_{12} \quad (6)$$

is the Raman detuning. The amplitudes E_L and E'_L , the phases $\phi_{1,2}$, and the relaxation parameters of the Λ -system determine the response of the Λ -system and the state of the photon field at any time instant $t > 0$.

The study of coherent information extracted in this way simultaneously answers the fundamental question of whether quantum information can be extracted using

a classical excitation, which, if weak, can only lead to the quasi-classical excitation of the photon field to a coherent state. As far as the TLA is concerned, this question should generally be answered in the positive way, as it follows, for example, from the results of [10]. This is due to the essentially quantum character of an atom, which can be viewed as a converter of the classical laser field.

In its pure form, the problem of the Λ -system–photon field channel can be considered only in the case of its single utilization since a reset of the channel involves the introduction of new quantum systems (reservoirs), which bring in new quantum information. A single extraction of quantum information is realized by a pulsed laser excitation, which entangles the initial state of the atom with the states of the photon field. Apart from the obvious qualitative differences between a Λ -system and a TLA, this process is analogous to the atom–field channel considered in [5]. Thus, the initial analysis of this problem naturally leads us to the problem of the pulsed excitation that should be applied at times shorter than the radiative lifetime to avoid decay-induced distortions and thereby provide the maximum flexibility of the control.

3.1. Calculation of the Coherent Information in the Case of Pulsed Excitation

In the case of pulsed excitation, the information is transferred from a specified initial state to a state of the photon field created due to the action of exciting laser pulses on the Λ -system. The input of the channel is the state at the time instant $t = 0$, specified by an arbitrary 2×2 density matrix of the form

$$\hat{\rho}_{in} = \sum \rho_{kl} |k\rangle\langle l|, \quad (7)$$

where the matrix ρ_{kl} , $k, l = 1, 2$ is positive-definite and the trace is equal to unity, $\rho_{11} + \rho_{22} = 1$. The initial state of the Λ -system is then given by the same matrix (7) viewed as a three-level system operator (for the considered class of initial states, we assume that the population of the excited state $|3\rangle$ is zero). In the rotating-wave approximation, the Liouvillian L does not contain any relaxation parameters if we restrict ourselves to laser pulses with durations $\tau_p \ll \gamma^{-1}$, where γ is the rate of radiative decay. Consequently, any pure initial state $\psi = c_1|1\rangle + c_2|2\rangle$ is transformed to a pure excited state of the form

$$\psi(0) = \begin{pmatrix} \Psi_1(0) \\ \Psi_2(0) \\ \Psi_3(0) \end{pmatrix} = \sum_{k=1}^3 (Uc)_k |k\rangle, \quad (8)$$

$$c = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \\ U_{31} & U_{32} \end{pmatrix}.$$

This state subsequently relaxes by emitting photons and becomes a metastable state of the form $\hat{\rho}_f = \sum_{k,l=1}^2 \rho_{fkl} |k\rangle\langle l|$. The latter state can also contain some information about the initial state, making the channel $\hat{\rho}_{\text{in}} \rightarrow \hat{\rho}_f$ another subject for consideration. However, the optimization of this channel over the exciting field is trivial: the optimum corresponds to no excitation. The nontrivial channel that is considered here involves the transfer of information to the photon field.

Photons are emitted exclusively by the excited state $|3\rangle$. If the interaction Hamiltonian preserves the total number of excitation quanta in the atom + field system, the dynamics can be solved exactly: it is reduced to the emission of a superposition of two photons corresponding to the transitions $|3\rangle \rightarrow |1\rangle$ and $|3\rangle \rightarrow |2\rangle$. The photons are described in the same fashion as in the case of the TLA except for the fact that one has to take into account the connection via the common excited level $|3\rangle$. In contrast to the TLA, the photon emitted by the Λ -system splits into a superposition of the two photons that correspond to its two transitions and oscillate at the respective frequencies ω_{13} and ω_{23} .

Thus, at any time instant $t \geq \gamma^{-1}$, τ_p , the state of the atom + field system is described by the wave function

$$\begin{aligned} \Psi(t) = & \psi_1(0)e^{-i\omega_1 t} |1\rangle|0\rangle + \psi_2(0)e^{-i\omega_2 t} |2\rangle|0\rangle \\ & + \psi_3(0)e^{-i(\omega_3 + \Lambda)t - \gamma t/2} |3\rangle|0\rangle \\ & + \psi_3(0)\sqrt{1 - e^{-\gamma t}} (\alpha_1 |1\rangle|\psi_{13}\rangle + \alpha_2 |2\rangle|\psi_{23}\rangle). \end{aligned} \quad (9)$$

Here, ω_1 , ω_2 , and ω_3 are the eigenfrequencies of the energy levels; Λ_{13} and Λ_{23} are the radiative frequency shifts due to the transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$; γ_{13} and γ_{23} are the rates of the radiative decay from level 3 to levels 1 and 2, respectively; $\gamma = \gamma_{13} + \gamma_{23}$ is the total decay rate of the excited state 3; $\Lambda = \Lambda_{13} + \Lambda_{23}$ is the total frequency shift; $\alpha_1 = \gamma_{13}/\gamma$ and $\alpha_2 = \gamma_{23}/\gamma$ describe the distribution the radiative decay over the two considered dipole transitions; $|0\rangle$, $|\psi_{13}\rangle$, and $|\psi_{23}\rangle$ are, respectively, the vacuum state of the photon field and the states with a single photon at the frequency ω_{13} and ω_{23} . The first three terms describe a superposition of the atomic states in the absence of photons; the last term, a superposition of two photon states that are entangled with the corresponding levels 1 and 2 of the Λ -system. We ignore the decoherence in this approach as the photon wave functions $|\psi_{13}\rangle$ and $|\psi_{23}\rangle$ are assumed to be known exactly due to the fact that we impose no restrictions on the way the photon signal can be used [11]. As opposed to the TLA–photon field channel, where the relevant set of photon states constitutes a two-level system, here it consists of three states, $|0\rangle$, $|\psi_{13}\rangle$, and $|\psi_{23}\rangle$, which are viewed, similar to [5], as the basis states, in terms of which the subsequent analysis of the channel

transformation is performed. It is easy to verify that the partial density matrices of the atom and the field that are associated to the wave function (9) correspond to the description of the radiative decay in terms of the relaxation dynamics of open systems. As for the fast-oscillating exponents before the atomic wave functions $|1, 2, 3\rangle$, one can easily see that, being associated to wave functions, they do not affect the information characteristic and therefore can be neglected during their calculation.

The function (9) describes the isometric transformation V : atom \rightarrow atom + field, i.e., the transformation $\psi(0) \rightarrow \Psi(t)$ of the form $\Psi(t) = \sum_{k\alpha,l} V_{k\alpha,l} \psi_l(0) |k\rangle|\alpha\rangle$, where $k, l = 1, 2, 3$ number the atomic states, and α numbers the states of the photon field. Expressing $\psi_l(0)$ through the initial state of the Λ -system before the application of the laser pulses, we obtain an isometric mapping of the GTLS to a state of the atom + field system described by the matrix $W_{k\alpha,m} = \sum_l V_{k\alpha,l} U_{lm}$. The corresponding superoperator of the GTLS–photon field channel has the form $\mathcal{C} = \sum \hat{s}_{mn} \langle m| \odot |n\rangle$ [5], where \odot is the substitution symbol that should be replaced by the density matrix to be transformed, $\hat{\rho}_{\text{in}}$, and \hat{s}_{mn} is the 2×2 matrix of the photon-field operators, which are represented by 3×3 matrices with the matrix elements

$$(\hat{s}_{mn})_{\alpha\beta} = \sum_{k=1}^3 W_{k\alpha,m} W_{k\beta,n}^*. \quad (10)$$

Here, the summation is performed over all three atomic states, and the output indices number the photon states $0, \psi_{13}$, and ψ_{23} .

If the Λ -system is excited by two rectangular pulses with a negligible detuning and a duration τ_p , the matrix U defining the initial atomic state depends on three parameters: $\theta = \sqrt{\Omega_1^2 + \Omega_2^2} \tau_p$ is the total action angle of the two pulses; $\chi = \arctan(\Omega_1/\Omega_2)$ is the angle describing the distribution of the field intensity over the pulses; ϕ is the relative phase of the pulses. By varying these parameters we can optimize the amount of information that is transferred from the initial state to the states of the photon field. In the case of the maximum-entropy initial state $\hat{\rho}_{\text{in}} = \hat{I}/2$, which provides the maximum amount of information for a certain choice of the laser parameters (one can show this using the symmetry arguments), the coherent information does not depend on the parameters χ, ϕ , which can therefore be neglected. The corresponding set of operators \hat{s}_{mn} has the form

$$\hat{s}_{11} = \begin{pmatrix} \cos^2 \frac{\theta}{2} + e^{-\gamma t} \sin^2 \frac{\theta}{2} & -\frac{i}{2} \sqrt{\alpha_1} \sqrt{1 - e^{-\gamma t}} \sin \theta & 0 \\ \frac{i}{2} \sqrt{\alpha_1} \sqrt{1 - e^{-\gamma t}} & \alpha_1 (1 - e^{-\gamma t}) \sin^2 \frac{\theta}{2} & 0 \\ 0 & 0 & \alpha_2 (1 - e^{-\gamma t}) \sin^2 \frac{\theta}{2} \end{pmatrix}, \tag{11}$$

$$\hat{s}_{12} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ i \sqrt{\alpha_2} \sqrt{1 - e^{-\gamma t}} \sin \frac{\theta}{2} & 0 & 0 \end{pmatrix}, \quad \hat{s}_{21} = \begin{pmatrix} 0 & 0 & -i \sqrt{\alpha_2} \sqrt{1 - e^{-\gamma t}} \sin \frac{\theta}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{s}_{22} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

After defining the operators \hat{s}_{mn} , which determine the superoperator \mathcal{C} of the considered channel, we carry out the rest of the calculation using the general formulas given in [5].

4. RESULTS OF CALCULATION

Figures 1 and 2a show the dependence of the coherent information in a symmetric Λ -system ($\gamma_{13} = \gamma_{23}$) on the relevant parameters of the considered channel. The total action integral $\theta = \Omega\tau_p$ is one of such parameters. In the case of a symmetric system and the initial state $\hat{\rho}_{in} = \hat{I}/2$, the coherent information does not depend on the distribution of the driving intensity over the two laser fields. Input density matrices of the general form do not provide the maximum amount of information; moreover, in this case, the coherent information depends on the intensity distribution of the exciting pulses, i.e., the parameter χ . Figure 1a shows the coherent

information at $t \rightarrow \infty$ as a function of the angle χ in the case of the diagonal input density matrix $\hat{\rho}_{in} = \begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}$. Figure 1b shows the coherent information as a function of the dimensionless time γt in the case of the maximum-entropy input density matrix $\hat{\rho}_{in} = \hat{I}/2$. Figure 2 shows the coherent information of the symmetric system at $t \rightarrow \infty$ as a function of the input density matrix $\hat{\rho}_{in}$.

It follows from these results that, in the case of a symmetric Λ -system, the maximum of the coherent information is achieved with the following parameters:

$$\hat{\rho}_{in} = \frac{\hat{I}}{2}, \quad t \rightarrow \infty, \quad \theta = \pi.$$

The first condition ensures that the amount of quantum information at the input is maximum; the second, that it

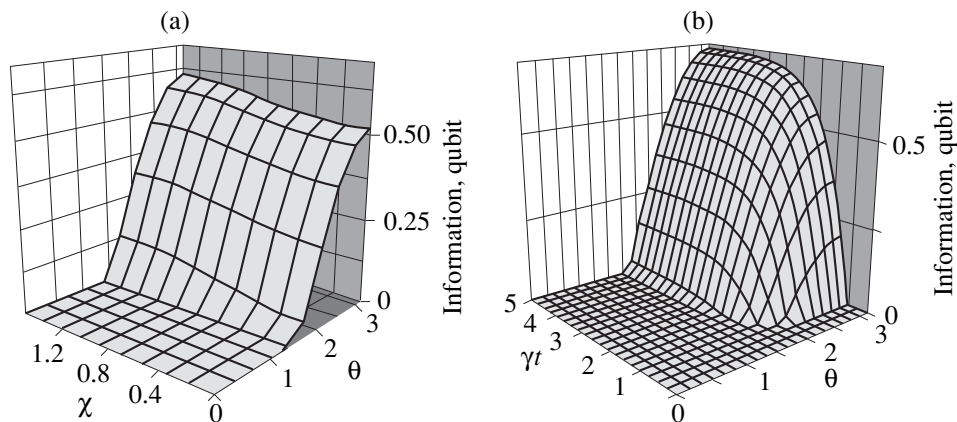


Fig. 1. Coherent information in a symmetric Λ -system (a) as a function of the total action integral $\theta = \Omega\tau_p$ and the angle χ describing the distribution of the driving field amplitudes over the levels 1 and 2 with $t \rightarrow \infty$ (the input density matrix is diagonal with the matrix element $\rho_{11} = 1/4$) and (b) as a function of the total action integral $\theta = \Omega\tau_p$ and the dimensionless time γt (the input density matrix is the maximum-entropy state $\hat{I}/2$).

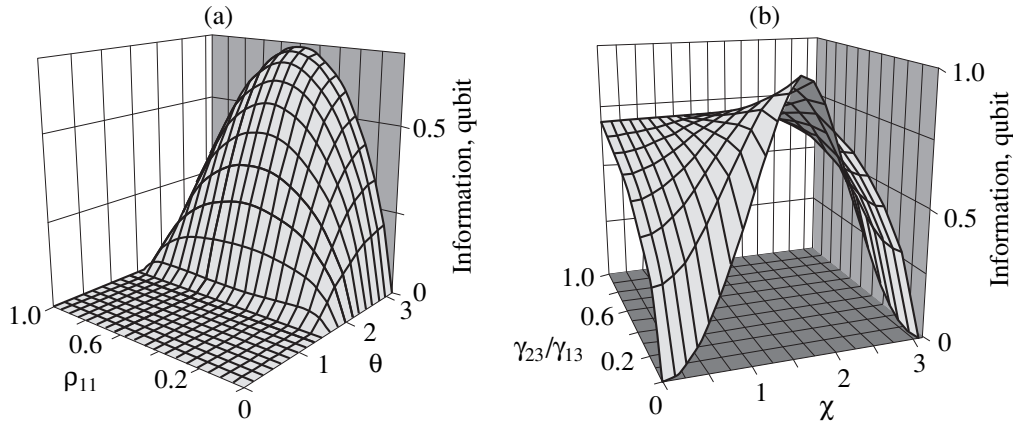


Fig. 2. (a) Coherent information in a symmetric Λ -system as a function of the total action integral $\theta = \Omega\tau_p$, and the matrix element ρ_{11} of the diagonal density matrix with $t \rightarrow \infty$. (b) Coherent information as a function of the system asymmetry γ_{23}/γ_{13} and the driving field distribution angle χ with $t \rightarrow \infty$ for $\hat{\rho}_{in} = \hat{I}/2$.

is maximally transferred to the photon field; and the third, that the population from the ground state is maximally transferred to the excited one. The corresponding maximum amount of quantum information, i.e., the potentially attainable information capacity of the channel, is $I_c = 0.6887$.

Figure 2b shows the dependence of I_c on the asymmetry degree γ_{23}/γ_{13} and the driving-intensity distribution angle χ in the case of a fully emitted photon, i.e., $\gamma t \rightarrow \infty$, the optimal value of $\theta = \pi$, and $\hat{\rho}_{in} = \hat{I}/2$. The maximum, $I_c = 1$ qubit, is reached at $\chi = \pi/2$ and $\gamma_{23}/\gamma_{13} = 0$, which corresponds to the reduction of the Λ -system to a two-level system. The optimization over the relative phase of the pulses ϕ is redundant for any values of the other parameters since this corresponds to a variation in the coding method but not the amount of information.

5. CONCLUSIONS

Thus, the utilization of a symmetric Λ -system for the transfer of information to the photon field somewhat reduces the information capacity of the channel with respect to the emission of a two-level system. This deficiency in the information capacity can be viewed as the commission for the radiative stability of the stored qubit and the advantages in the radiative manipulation of its states. Therefore, the photon field cannot, even in principle, be used for quantum computation as an equivalent of the information qubit stored in the ground state of the Λ -system. Nevertheless, as far as multiqubit operations are concerned, the information loss amounts to only 31%, which demonstrates the possibilities for a fairly efficient utilization of the photon field for the physical conversion of coherent quantum information. The performed analysis also provides a general idea about the information losses intrinsically related to the

extraction of the information about the ground state of the Λ -system by the methods of laser spectroscopy.

ACKNOWLEDGMENTS

This research was supported by the ‘‘Nanotechnologies’’ and ‘‘Fundamental Metrology’’ programs of the Ministry of Science and Technologies of Russian Federation.

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- The complex conjugation, which is introduced here and was absent in [4], is required for the invariance of the considered representation with respect to rotations in the subspaces corresponding to degenerate eigenvalues of the density matrix. For real density matrices ρ_{in} with nondegenerate spectra, this refinement is unnecessary.
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- To take into account the restrictions due to the actual geometry of the photodetector and its finite time resolution, one should introduce a corresponding transfer function. Obviously, it is here that lies the border between the description of the atom + field system dynamics in terms of a unitary transformation and a relaxation random process.